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On the problem of heat transfer in phase-change materials for small Stefan numbers

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Abstract—The problem of solidification of an infinite liquid slab by linear convection cooling from the adjacent air is considered. It is assumed that at the initial moment the slab has a critical temperature and that on one side the air temperature has diurnal fluctuations of relatively small amplitude. The solution is found in the form of power series of small parameters. The first three terms of each series are obtained. This permits construction of a simple formula for the total solidification time. Evaluation of the accuracy of the solution based on the integral heat balance equation is suggested. Some particular cases of the problem are considered on the assumption that one side of the slab is insulated. One of these cases is the simplest Stefan problem which has an exact solution. Comparison with this solution is made.

1. INTRODUCTION

RECENTLY, the application of phase-change material (PCM) to problems of thermal energy storage has increased greatly. Thus simple analytic solutions of heat transfer problems with phase changes now have practical significance.

To a great extent, the processes of heat transfer in PCM are characterized by the so-called Stefan number. A low value of the Stefan number is characteristic of a large class of practical problems in which latent heat of phase change plays the principal role. It is natural to use this feature to construct approximate analytical solutions to such problems. To achieve this aim, some authors either make speculations based on the results of the numerical solution (e.g. [1]), or they begin by converting the governing equation to the new independent variables, which complicates these equations (e.g. [2, 3]). Thorough discussion and review of the Stefan problem for small Stefan numbers is given in refs. [3–6].

It seems that in certain cases a solution can be obtained more simply by a direct expansion into a power series of the Stefan number and, perhaps, of some other small parameters. Furthermore, for the evaluation of the accuracy of these solutions, relatively simple and sufficiently reliable formulae can be obtained.

We shall try to demonstrate such a technique for a rather general and, at the same time, not very complicated problem with applications to PCM. In this problem, external cooling (or heating) input has a periodic component (in time). The problem is simplified for methodological purposes, particularly with respect to the geometry and choice of initial conditions.

2. FORMULATION OF THE PROBLEM

We consider a solidification problem of an infinite liquid PCM slab whose initial temperature is a critical one. The slab is cooled from both sides by linear convective heat transfer from the adjacent fluid (air, in particular). Moreover, it is assumed that one face of the slab is exposed to solar radiation. (We put the origin of coordinate $x = 0$ at this boundary of the slab. Then the opposite boundary of the slab is $x = d$.) For simplicity, we assumed that the fluctuations of the solar influx and of the air temperature are sinusoidal, but the amplitude of the fluctuations is relatively small and therefore not sufficient to prevent solidification.

It is easy to see that, in this problem, the solidification of each side of the slab is independent and therefore can be considered separately. For the exposed side, one has the following problem:

$$\left.\begin{aligned}
&\frac{\partial T(x,t)}{\partial t} = \alpha\frac{\partial^2 T(x,t)}{\partial x^2};\\
&\delta(t) > x > 0,\quad \theta \geqslant t > 0\\
&\delta(0) = 0,\quad K\left.\frac{\partial T}{\partial x}\right|_{x=0} = h(T|_{x=0} - T_{\rm in});\\
&K = \rho c\alpha\\
&T|_{x=\delta} = T_{\rm cr},\quad \rho L\dot{\delta} = K\left.\frac{\partial T}{\partial x}\right|_{x=\delta};\\
&(\dot{\ }) = \frac{\rm d}{{\rm d}t}(\ )\\
&T_{\rm in}(t) = T_{\rm a} + A\sin\Omega t \leqslant T_{\rm cr}
\end{aligned}\right\} \tag{1}$$

It is assumed that α, ρ, c, h, L, T_{cr}, T_a, A and Ω are constant. The last inequality may be stronger than is necessary for the validity of the solution which will be obtained. It could be argued that $T_w < T_{cr}$ would be sufficient for a solution, but T_w is not known in advance. Note, that the linearized convective boundary condition at $x = 0$ is rather common.

One can show that this approximately allows for diurnal variations of the direct and scattered solar radiation. It evidently also accounts for the influence of

NOMENCLATURE

- A amplitude of external temperature fluctuations [°C]
- c solid PCM specific heat [kJ kg^{-1} °C^{-1}]
- d slab thickness [m]
- h heat transfer (or film) coefficient at the exposed slab boundary [kJ m^{-2} s^{-1} °C^{-1}]
- K solid PCM conductivity [kJ m^{-1} s^{-1} °C^{-1}]
- L PCM latent heat [kJ kg^{-1}]
- q heat flux [kJ m^{-2} s^{-1}]
- t time [s]
- T PCM temperature, (solid part) [°C]
- T_{cr} PCM critical temperature [°C]
- T_{in} transfer air inlet temperature at the exposed side of the slab [°C]
- T_a constant part of T_{in} [°C]
- T_w temperature at the boundary $X = 0$ [°C]
- ΔT characteristic change of temperature, (solid part) [°C]
- x coordinate, normal to the boundary of the slab [m]
- X characteristic linear scale [m].

Greek symbols

- α solid PCM thermal diffusivity [m^2 s^{-1}]
- δ the thickness of the solidified layer [m]
- ρ solid PCM density [kg m^{-3}]
- θ total solidification time [s]
- Θ characteristic time scale [s]
- Ω angular earth rotation velocity [s^{-1}].

Non-dimensional parameters

- a $A/\Delta T$
- Bi Biot number, hd/K
- St Stefan number, $c\Delta T/L$
- μ $\Delta T'/\Delta T$
- v h/h'
- ω $\rho LK\Omega/\Delta Th^2$.

The primed letters refer to the solidified layer which is developing at the opposite side of the slab $\delta' \ll x \ll d$.

a thin stratum of foreign material which is often placed on the surfaces of the slab for technical reasons. Therefore, h and T_{in} should be considered as effective 'heat transfer coefficient' and 'outer media temperature', respectively. If necessary, one calculates h and T_{in} as functions of the solar influx, wind and air temperature and some other parameters.

At $t = \theta$ the phase-change interfaces meet and the slab becomes completely solid. Hence

$$\delta(\theta)+\delta'(\theta) = d. \tag{2}$$

Hereafter, the primed letters refer to the solidified layer $\delta'(t) \leqslant x \leqslant d$ which is developing at the opposite side of the slab.

Note, that one of the objectives of this paper is to find an analytic solution for θ.

The formulation of the problem can be supplemented by the integral thermal energy (or heat) balance equation

$$-\int_0^t K\left.\frac{\partial T}{\partial x}\right|_{x=0} \mathrm{d}t+\int_0^\delta \rho c(T_{cr}-T)\,\mathrm{d}x + \rho L\delta = 0; \theta \geqslant t \geqslant 0 \tag{3}$$

which is a consequence of equations (1). Equation (3) will be employed for an estimation of the accuracy of the approximate solution for any t including $t = \theta$.

3. DIMENSIONALIZATION OF THE EQUATIONS

For convenience of the analysis and solution of the problem we shall convert (1)–(3) to non-dimensional variables (letters with bars) by the following relations:

$$x = X\bar{x}, \quad \delta = X\bar{\delta}, \quad t = \Theta\bar{t}, \quad T = T_{cr}-\Delta T\bar{T}. \tag{4}$$

We shall try to choose such values for X, θ and ΔT, which might be considered as characteristic scales.

We shall denote

$$\left.\begin{aligned} a = A/\Delta T, \quad \omega = \rho LK\Omega/h^2\Delta T, \\ v = h/h' \\ \mu = \Delta T'/\Delta T;\ \Delta T = T_{cr}-T_a, \\ \Delta T' = T_{cr}-T'_{in} \end{aligned}\right\} \tag{5}$$

assuming that h' and T'_{in} are constant.

Besides these non-dimensional parameters, we shall use conventional notation for Stefan and Biot numbers ($St = c\Delta T/L$; $Bi = hd/K$).

Let us put

$$X = K/h, \quad \Theta = \rho LK/h^2\Delta T = X^2/\alpha St \tag{6}$$

then equations (1)–(3) become (the bars are omitted)

$$\left.\begin{aligned} & St\frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T(x,t)}{\partial x^2};\ \delta(t) > x > 0, \\ & \tau \geqslant t > 0 \\ & \delta(0) = 0, \quad \left.\frac{\partial T}{\partial x}\right|_{x=0} = T|_{x=0} - 1 + a\sin\omega t \\ & T|_{x=\delta} = 0, \quad \dot{\delta} = -\left.\frac{\partial T}{\partial x}\right|_{x=\delta} \end{aligned}\right\} \tag{7}$$

$$\delta(\tau)+v\delta'(\tau') = Bi;\ \tau = \theta/\Theta, \quad \tau' = \mu\tau/v^2 \tag{8}$$

$$\int_0^t \left.\frac{\partial T}{\partial x}\right|_{x=0} \mathrm{d}t + St\int_0^\delta T\,\mathrm{d}x + \delta = 0;\ \tau \geqslant t \geqslant 0. \tag{9}$$

One can see from (7) that the body of the problem depends only upon the three non-dimensional parameters St, a and ω. The parameters Bi, ν and μ actually appear after solving the problem.

For example, we shall consider a concrete case of a Glauber salt used as a slab of PCM. Then one can put

$$
\begin{aligned}
\alpha &= 0.84\times10^{-6}\ \mathrm{m^2\ s^{-1}},\\
\rho &= 1.46\times10^{3}\ \mathrm{kg\ m^{-3}},\\
c &= 1.76\ \mathrm{kJ\ kg^{-1}\ {}^\circ C^{-1}}\\
L &= 2.51\times10^{2}\ \mathrm{kJ\ kg^{-1}},\\
T_{cr} &= 32^\circ\mathrm{C},\\
K &= 2.16\times10^{-3}\ \mathrm{kJ\ m^{-1}\ s^{-1}\ {}^\circ C^{-1}}\\
h &= 4\times10^{-2}\ \mathrm{kJ\ m^{-2}\ s^{-1}\ {}^\circ C^{-1}},\quad d = 0.1\ \mathrm{m},\\
A &= 5^\circ\mathrm{C}\quad \Omega = 10^{-4}\ \mathrm{s^{-1}}.
\end{aligned}
\tag{10}
$$

The values for the thermophysical parameters are taken from Solomon [4]. The temperature data seem plausible for desert conditions in summer.

From (5) and (6) we obtain

$$
a = 0.23,\quad \omega = 2.25,\quad X = 5.4\ \mathrm{cm},
$$

$$
\Theta = 6.25^{h}(St = 0.15,\ Bi = 1.85). \tag{11}
$$

The characteristic thickness and time obtained are in accordance with experiments (the order of magnitude of these values is right). Therefore, one can conclude that low values of St and a in (11) indicate the smallness of the terms which appear in (7) and (9) with these parameters.

4. SOLUTION OF THE PROBLEM

We seek a solution of (7) in the form of infinite power series in St and a:

$$
\left.
\begin{aligned}
T &= T_0(x,t) + St\,T_1(x,t) + aT_2(x,t) + \cdots\\
\delta &= \delta_0(t) + St\,\delta_1(t) + a\delta_2(t) + \cdots.
\end{aligned}
\right\} \tag{12}
$$

Substituting (12) into (7) one obtains the infinite succession of problems:

$$
\left.
\begin{aligned}
&\frac{\partial^2 T_0}{\partial x^2} = 0,\quad \left.\frac{\partial T_0}{\partial x}\right|_{x=0} = T_0|_{x=0} - 1\\
&\delta_0(0) = 0,\quad T_0|_{x=\delta_0} = 0\quad \dot{\delta}_0 = -\left.\frac{\partial T_0}{\partial x}\right|_{x=\delta_0}
\end{aligned}
\right\} \tag{13}
$$

$$
\left.
\begin{aligned}
&\frac{\partial T_0}{\partial t} = \frac{\partial^2 T_1}{\partial x^2},\quad \left.\frac{\partial T_1}{\partial x}\right|_{x=0} = T_1|_{x=0}\\
&\delta_1(0) = 0,\quad T_1|_{x=\delta_0} + \left.\frac{\partial T_0}{\partial x}\right|_{x=\delta_0}\delta_1 = 0,\\
&\dot{\delta}_1 = -\left.\frac{\partial T_1}{\partial x}\right|_{x=\delta_0}
\end{aligned}
\right\} \tag{14}
$$

$$
\left.
\begin{aligned}
&\frac{\partial^2 T_2}{\partial x^2} = 0,\quad \left.\frac{\partial T_2}{\partial x}\right|_{x=0} = T_2|_{x=0} + \sin\omega t\\
&\delta_2(0) = 0,\quad T_2|_{x=\delta_0} + \left.\frac{\partial T_0}{\partial x}\right|_{x=\delta_0}\delta_2 = 0,\\
&\dot{\delta}_2 = -\left.\frac{\partial T_2}{\partial x}\right|_{x=\delta_0}.
\end{aligned}
\right\} \tag{15}
$$

It should be noticed that the functions which refer to $x = \delta(t)$ were expanded in the Taylor series in the vicinity of $x = \delta_0(t)$.

Solving problems (13), (14) and (15) consecutively, one can find*

$$
\delta_0 = (1+2t)^{1/2} - 1,\quad T_0 = (\delta_0 - x)\dot{\delta}_0 \tag{16}
$$

$$
\left.
\begin{aligned}
\delta_1 &= -\frac{\delta_0^2\delta_0}{2(1+\delta_0)}\left(1+\frac{\delta_0}{3}\right)\\
T_1 &= -\left(\frac{\delta_0^3\dddot{\delta}_0}{1+\delta_0}\right)\frac{x+1}{3} + (\delta_0\ddot{\delta}_0)\frac{x^2}{2} - \ddot{\delta}_0\cdot\frac{x^3}{6}
\end{aligned}
\right\} \tag{17}
$$

$$
\left.
\begin{aligned}
\delta_2 &= -(1-\cos\omega t)/\omega(1+2t)^{1/2}\\
T_2 &= \frac{\sin\omega t}{(1+2t)^{1/2}}\left[x+1-(1+2t)^{1/2}\right]\\
&\quad - \frac{1-\cos\omega t}{\omega(1+2t)^{1/2}}(x+1).
\end{aligned}
\right\} \tag{18}
$$

The validity of equations (16)–(18) can easily be proven by substituting them into (13)–(15).

For an indirect estimate of the accuracy of the solution, we shall employ (9). Evidently, the substitution of the infinite series (12) into (9) must lead to an identity because (9) is a consequence of (7).

However, the substitution of the truncated series (12) into (9) does not yield zero [because of the presence in (9) of the term with the factor St and with δ as an upper limit of the integral], but simply an error at which equation (9) is not satisfied.

If we confine ourselves to terms with first powers of St and a, and divide the result by δ_0, we obtain the following expression for the relative error:

$$
\varepsilon(t) = \frac{St}{\delta_0}\int_0^{\delta_0}(St\,T_1 + aT_2)\,\mathrm{d}x. \tag{19}
$$

Then, using (16)–(18), an elementary transformation yields:

$$
\left.
\begin{aligned}
\frac{1}{\delta_0}\int_0^{\delta_0} T_1\,\mathrm{d}x &= -\frac{\delta_0^2(3\delta_0^2+15\delta_0+20)}{24(1+\delta_0)^4}\\
\frac{1}{\delta_0}\int_0^{\delta_0} T_2\,\mathrm{d}x &= \frac{1}{2(1+\delta_0)}\left[\delta_0\sin\omega t\right.\\
&\quad\left.+\frac{\delta_0+2}{\omega(1+\delta_a)^2}(1-\cos\omega t)\right].
\end{aligned}
\right\} \tag{20}
$$

To find θ, we have to solve equation (8) for τ. Substituting (12) into (8), we shall seek τ in the same form of a power series:

$$
\tau = \tau_0 + St\cdot\tau_1 + a\tau_2 + \ldots. \tag{21}
$$

* Note, that expressions (17) are valid even in the case when a is not small. But then

$$
\delta_0(t) = [1 + 2t - 2a(1-\cos\omega t)/\omega]^{1/2} - 1
$$

Maintaining the accuracy of the solution, one can truncate series (21) after the first three terms. Equating the terms with like powers of St and a, one arrives at a quadratic equation for τ_0 and two linear algebraic equations for τ_1 and τ_2. The solutions of these equations are

$$\left.\begin{aligned}\tau_0 &= \frac{1}{1-\mu}\left\{\frac{v^2-1}{2}+\frac{(Bi+v+1)^2}{1-\mu}\left[\frac{1+\mu}{2}\right.\right.\\ &\quad\left.\left.-\left(\mu-\frac{(1-\mu)(v^2-\mu)}{(Bi+v+1)^2}\right)^{1/2}\right]\right\}\\ \tau_1 &= -\beta[\delta_1(\tau_0)+v\mu\delta_1(\tau_0\mu/v^2)],\\ \tau_2 &= \frac{\beta(1+\cos\omega\tau_0)}{\omega(1+2\tau_0)^{1/2}}\\ \beta &= [(1+2\tau_0)^{-1/2}+\mu(v^2+2\mu\tau_0)^{-1/2}]^{-1}.\end{aligned}\right\} \quad (22)$$

Note, that in the expression for τ_0 the sign in front of the radical is specified by the trivial condition: $\tau_0 \to 0$ if $Bi \to 0 (d \to 0)$.

5. SOME SIMPLER VARIANTS OF THE PROBLEM

From (22) one can obtain formulae for the rather important case of a slab which is insulated at $x = d$. This can be done in two ways. We can either put $h' = 0$, which means finding a limit of (22) for $v \to \infty$; or, what is somewhat simpler, to put $T'_{in} = T_{cr}$ and $h' \to \infty$ which means finding a limit of (22) for $v \to 0, \mu \to 0$. Evidently, both ways give the same result

$$\left.\begin{aligned}\tau_0 &= Bi(Bi+2)/2, \quad \tau_1 = B_i^2(Bi+3)/6(Bi+1)\\ \tau_2 &= (1-\cos\omega\tau_0)/\omega, \quad \delta_0(\tau_0) = Bi.\end{aligned}\right\} \quad (23)$$

In the case under consideration, it is convenient to write the characteristic scales in the form

$$X = d/Bi, \quad \Theta = d^2/\alpha St\, Bi^2 \quad (24)$$

Then, multiplying (21) by the Θ term by term and using (23) after an elementary transformation, we obtain

$$\theta = \frac{d^2}{\alpha St}\left[\frac{1}{2}+\frac{St}{6}\frac{Bi+3}{Bi+1}+\frac{1}{Bi}+\frac{a(1-\cos\omega\tau_0)}{\omega Bi^2}\right]. \quad (25)$$

In accordance with (19) and (20) in which $\delta_0(\tau_0) = Bi$, the energetics evaluation is now

$$\begin{aligned}\varepsilon(\theta) &= -[(St\,Bi)^2(3Bi^2+15Bi+20)/24(Bi+4)^4]\\ &\quad + aSt[Bi\sin\omega\tau_0+(Bi+2)\\ &\quad \times(1-\cos\omega\tau_0)/\omega(Bi+1)^2]/2(Bi+1). \quad (26)\end{aligned}$$

* Other approximate expressions for θ for both simplified variants ($Bi < \infty$ and $Bi = 0$) when $A = a = 0$ have been obtained and expanded upon in the series of papers by Solomon (e.g. [1, 7]).

Concerning the solutions for δ_i and $T_i (i = 0, 1, 2)$, they remain in form of (16), (17) and (18).

It is interesting to compare an approximate solution, obtained by the expansion into series, with the exact solution. For this, one can consider the simplest variant of the problem which has the well-known exact solution found by Stefan. In this variant, the temperature at $x = 0$ is $T_w = \text{const} < T_{cr}, (\theta \geqslant t \geqslant 0)$. One can arrive at this variant by allowing $h \to \infty$ $(Bi \to \infty)$ and putting $a = 0$, $T_a = T_w$ in (25) and (26).

We easily obtain*

$$\theta = \frac{d^2}{\alpha St}\left(\frac{1}{2}+\frac{St}{6}\right), \quad \varepsilon(\theta) = -\frac{1}{8}St^2. \quad (27)$$

The exact Stefan solution for δ is

$$\delta_{ex}(t) = 2\lambda\sqrt{\alpha t}(\sqrt{\pi}\lambda\, e^{\lambda^2}\operatorname{erf}\lambda = St). \quad (28)$$

The transcendental equation in parentheses serves for finding λ. If St is small, λ also has to be small. Using this we transform the equation to the form

$$\frac{1}{4\lambda^2} = \frac{1}{St}\left(\frac{1}{2}+\frac{St}{6}-\frac{1}{45}St^2+\cdots\right). \quad (29)$$

As a consequence of the first equation in (28), we have $\theta_{ex} = d^2/4\alpha\lambda^2$. From this and (29) one can see that the expression for θ in (27) coincides with the exact Stefan solution with the accuracy of the term at St to the first power.

With the help of (29) we arrive at an evaluation of the relative error of the approximate solution more rigorous than (27):

$$\frac{\theta_{ex}-\theta}{\theta_{ex}} \approx -\frac{2}{45}St^2. \quad (30)$$

One can see that this evaluation gives a result which is about three times smaller in absolute value than the energetics evaluation [second expression in (27)].

In conclusion, we shall consider a case where the heat flux which is absorbed by the exposed boundary of the slab is assumed given and constant. The opposite boundary is again insulated. Thus, only the boundary condition at $x = 0$ will be different from (1):

$$q = K\left.\frac{\partial T}{\partial x}\right|_{x=0} = \text{const}\ (\theta \geqslant t \geqslant 0). \quad (31)$$

In this variant it is convenient to specify the characteristic scales by the relations

$$X = d, \quad \Theta = d^2/\alpha St, \quad \Delta T = qd/K. \quad (32)$$

It turns out that it would be easier to solve this variant from scratch. Unlike the above, we found a solution with accuracy of the second powers of St. The method of solving is the same. Therefore, we present here only the final result

$$\left.\begin{aligned}\delta &= t-\frac{t^2}{2}St+\left(\frac{t^3}{3}-t\right)St^2 \\ T &= t-x+\left(\frac{x^2}{2}-t^2\right)(1-tSt)St \\ \theta^{(2)} &= \frac{d^2}{\alpha St}\left(1+\frac{1}{2}St+\frac{7}{6}St^2\right), \\ \varepsilon^{(2)}(\theta) &\approx \frac{13}{12}St^3.\end{aligned}\right\} \quad (33)$$

The superscripts show the largest power of the St in the expansion.

If we confine ourselves to the first power of St, the last term of the expression for $\theta^{(2)}$ in (33) can serve as an estimate of the error of $\theta^{(1)}$. Then

$$\frac{\theta^{(1)}-\theta^{(2)}}{\theta^{(2)}} \approx -\frac{7}{6}St^2. \quad (34)$$

Due to (18) and (33) we have

$$\varepsilon^{(1)}(\theta) \approx -\frac{2}{3}St^2. \quad (35)$$

One can see that in this case the direct evaluation of the error (34) gives a value nearly twice the absolute value of the energetics estimation (35).

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SUR LE PROBLEME DU TRANSFERT THERMIQUE DANS LES MATERIAUX AVEC CHANGEMENT DE PHASE AUX FAIBLES NOMBRES DE STEFAN

Résumé—On considère le problème de la solidification d'une couche infinie liquide, par convection linéaire d'air adjacent. On suppose qu'à l'instant initial la couche a une température critique et que sur un coté la température de l'air a une fluctuation diurne d'amplitude relativement faible. La solution trouvée est sous la forme d'une série puissance de petits paramètres. Les trois premiers termes de chaque série sont obtenus. Cela permet la construction d'une formule simple pour le temps total de solidification. On évalue la précision de la solution basée sur l'équation du bilan thermique intégral. Quelques cas particuliers du problème sont considérés pour une face de la couche isolée. L'un de ces cas est le problème de Stefan, le plus simple, qui a une solution exacte. On fait la comparaison avec cette solution.

WÄRMEÜBERTRAGUNG IN MATERIALIEN MIT PHASENÜBERGANG BEI KLEINEN STEFAN-ZAHLEN

Zusammenfassung—Es wird das Problem der Erstarrung einer unendlich großen Flüssigkeitsscheibe mittels linearer Konvektionskühlung durch die angrenzende Luft behandelt. Es wird angenommen, daß sich die Scheibe zu Anfang auf der kritischen Temperatur befindet und daß die Lufttemperatur auf der einen Seite täglichen Schwankungen mit relativ kleiner Amplitude unterliegt. Die Lösung wird in Form von Reihenentwicklungen kleiner Parameter dargestellt. Die ersten drei Terme jeder Reihenentwicklung werden betrachtet. Dies gestattet die Konstruktion einer einfachen Formel für die gesamte Erstarrungszeit. Es wird eine Abschätzung der Genauigkeit der Lösung, basierend auf der integralen Wärmebilanzgleichung, aufgezeigt. Einige Sonderfälle des Problems werden für die Abnahme einer einseitig wärmegedämmten Flüssigkeitsscheibe behandelt. Einer dieser Fälle ist das einfachste Stefan-Problem, wofür eine exakte Lösung vorliegt. Ein Vergleich mit dieser Lösung wurde vorgenommen.

К ВОПРОСУ О ТЕПЛОПЕРЕДАЧЕ С ИЗМЕНЕНИЕМ ФАЗОВОГО СОСТОЯНИЯ В МАТЕРИАЛАХ, ИМЕЮЩИХ МАЛЫЕ ЧИСЛА СТЕФАНА

Аннотация—Рассмотрена задача о затвердевании расплавленного вещества сохраняющего форму бесконечной пластины и имеющего в начальный момент критическую температуру. Предполагается, что пластина охлаждается с обеих сторон за счет конвекции соприкасающегося с ней воздуха, причем с одной из сторон температура этого воздуха имеет суточные колебания относительно малой амплитуды. Решение отыскивается в форме рядов по степеням безразмерных малых параметров задачи. Для каждого из степенных рядов получены первые три члена. Этого достаточно, чтобы построить простую формулу для времени полного затвердевания пластины. Предложена методика оценки точности решения, основанная на рассмотрении интегрального уравнения баланса тепла в пластине. Рассмотрены некоторые частные случаи задачи в предположении что одна из сторон пластины теплоизолирована. Один из этих случаев является простейшей задачей Стефана, имеющей точное решение, которое сравнивается с полученным приближенным решением.